HYDRODYNAMICS AND HEAT EXCHANGE IN A LIQUID FILM IN THE ENTRANCE SECTION WITH ALLOWANCE FOR FRICTIONAL FORCES AT THE INTERFACE

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The influence of the entrance section on the hydrodynamics and heat exchange in a liquid layer on a rotating surface is studied with allowance for frictional forces at the interface.

The hydrodynamics and heat exchange in a liquid layer on a rotating spiral surface in the absence of wave formation and with allowance for frictional forces at the interface is studied in [1] by the method of integral equations. In [2] these same problems are studied by the method of [3] without allowance for frictional forces at the interface. The hydrodynamics and heat exchange in a liquid film in the entrance section are studied in the present paper with allowance for the shear stress at the liquid film—gas surface. The assumptions relative to the physical statement of the problem are the same as in [2].

The system of equations describing the hydrodynamics and heat exchange in a liquid film on a rotating spiral surface is the following:

$$\begin{gathered} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=F_x-\frac{1}{\rho}\cdot\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2},\\ -\frac{u^2}{R(x)}=F_y-\frac{1}{\rho}\cdot\frac{\partial P}{\partial y},\\ \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\\ u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}. \end{gathered} \tag{1}$$

The influence of the gas stream on the flow of a thin liquid layer is accomplished through the tangential forces at the interface; i.e., with y = H(x) we have

$$\frac{du}{dy}=\frac{\tau_0}{\mu}=B. \tag{2}$$

We introduce the dimensionless shear stress, using the physical parameters Re, Ga, El, and Γ of the flow:

$$\overline{B}=\mathrm{Ga}\,\mathrm{Re}^{-1}E1^2\Gamma,$$

where

$$\mathrm{Re}=\frac{3q}{\nu},\ \mathrm{Ga}=\frac{\omega^2Ah_0^3}{\nu^2},\ E1=\frac{\delta_W}{h_0},\ \Gamma=\frac{3\tau_0}{\rho\omega^2Ah_0}.$$

The thickness δ_W of the liquid film, obtained from the solution of the system of equations (1) in the stabilization section, has the form

$$\delta_W=h_0\sqrt[3]{\frac{\mathrm{Re}}{\mathrm{Ga}}\left(1-\frac{\overline{B}}{2}\right)}. \tag{3}$$

In accordance with Eq. (3), the projections of the mass forces onto the coordinate axes take the form

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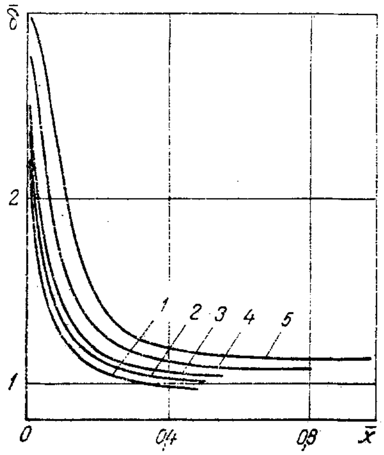


Fig. 1. Dependence of dimensionless film thickness on dimensionless length of spiral for Re = 300, E5 = 0.1, and E1 = 1/3; 1) Γ = −0.1; 2) 0; 3) 0.1; 4) 0.3; 5) 0.5. $\bar{\delta} = \delta/\delta_w$, $\bar{x} = x/\delta_w\text{Re}$.

$$\bar{F}_x = 9\left(1 - \frac{\bar{B}}{2}\right)\frac{\theta^2+1}{\theta^2+2} \pm \frac{6\text{Ga}^{1/2}E5^{1/2}E1^2}{\text{Re}}\bar{v},$$
$$\bar{F}_y = -\frac{9\text{Ga}E1^3}{\text{Re}^2}\theta\frac{\theta^2+1}{\theta^2+2} \mp \frac{6\text{Ga}^{1/2}E5^{1/2}E1^2}{\text{Re}}\bar{u}. \quad (4)$$

The boundary conditions for the system of equations (1) are the following:

$$\begin{aligned} &\text{for } y = 0, u = v = 0, T = T_w, \\ &\text{for } y = H(x), T = T_{fi}, \\ &\text{for } x = 0, T = T_w, \end{aligned} \quad (5)$$

where T_w and T_{fi} are the temperatures at the spiral surface and at the surface of the liquid film.

To study the development of the velocity profile and thickness of the liquid film in the initial section and of the heat flux at the spiral surface one must solve the system of equations (1) with the boundary conditions (2) and (5).

The procedure for solving such a system in the presence of shear stress at the liquid film—gas interface is analogous to that for solving the given problem in the absence of a gas interaction with the liquid film, which was described in detail earlier [2]. Therefore, we present the final results of the solution of the problem stated above.

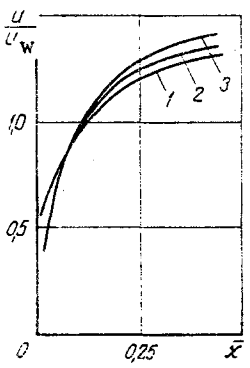
As seen from Figs. 1 and 2, the dimensionless shear stress has an important influence both on the development of the film thickness and on the development of the surface velocity.

The development of the velocity profile in the liquid film at different distances from the entrance is shown in Fig. 3 for Re = 300, E5 = 0.1, and E1 = 1/3; the solid lines are for Γ = 0 and the dashed lines for Γ = 0.1. With an increase in the number Γ the velocity profile becomes fuller than for gravitational flow of a liquid film. A similar pattern occurs in the wave flow of a liquid film interacting with a gas stream [4].

The average coefficient of heat transfer in the entrance section of a spiral heat exchanger is determined from the equation

$$\beta = \frac{1}{L}\int_0^L a\left(\frac{\partial T}{\partial y}\right)_{y=0} dx = \frac{1}{L}\left[\int_0^{H(x)} uTdy\right]_{x=L} = \frac{1}{L}HT. \quad (6)$$

The characteristic form of the dependence of the quantity HT^2 on the dimensionless length of the spiral for different values of the hydrodynamic parameters is shown in Fig. 4a–e. It is seen from the figure that large values of HT^2 correspond to large values of E1, E5, Re, Pr, and Γ.



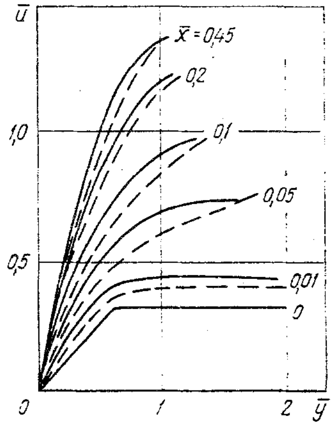


Fig. 2 Fig. 3

Fig. 2. Dependence of dimensionless surface velocity on dimensionless length of spiral for Re = 300, E5 = 0.1, and El = 1/3; 1) Γ = −0.1; 2) 0; 3) 0.5. $\bar{x} = x/\delta_w Re$.

Fig. 3. Dependence of dimensionless velocity on transverse coordinate at different distances from the entrance for Re = 300, E5 = 0.1, El = 1/3, and Γ = 0 and 0.1.

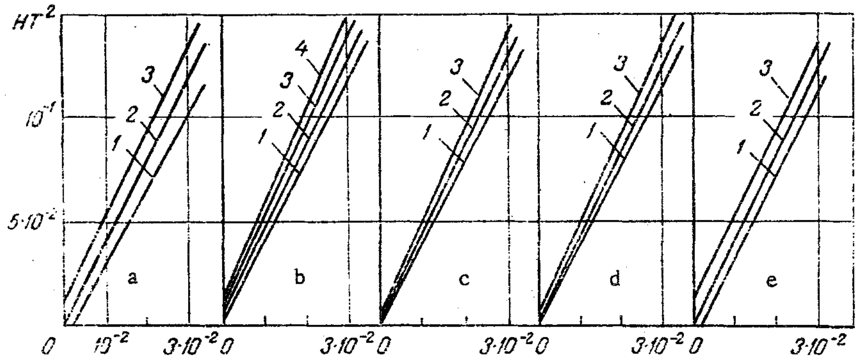


Fig. 4. Dependence of HT^2 on dimensionless length of spiral for a) Re = 100, E5 = 0.1, El = 0.1; 1) Pr = 10; 2) 30; 3) 100; b) Re = 100, E5 = 0.1, Pr = 30; 1) El = 0.1; 2) 0.4; 3) 1; 4) 1.6; c) Re = 100, El = 0.1, Pr = 30; 1) E5 = 0.1; 2) 0.5; 3) 1; d) E5 = 0.1, El = 0.1, Pr = 30; 1) Re = 100; 2) 500; 3) 1000; e) Re = 100, El = 0.5, E5 = 0.1, Pr = 30; 1) Γ = 0.5; 2) 0; 3) −0.3.

The results of a numerical solution of the system (1) with the boundary conditions (2) and (5), partially represented in Fig. 4, are approximated with an accuracy of ±10% by the following equation:

$$\beta = \frac{0.577 u_w^{1/2} a^{1/2}}{L^{1/2}} \left[\left(3.9 + 0.75E1 + 0.9E5 + 0.001\,\mathrm{Re} + 0.007\,\mathrm{Pr} + 0.005\frac{E1\delta_w \mathrm{Re}\,\mathrm{Pr}}{L}\right)(1 + 0.1\Gamma)\right]^{1/2}. \quad (7)$$

Equation (7) can be recommended for the calculation of heat exchange in a liquid film with whose surface a gas stream interacts; the latter is allowed for by the number Γ. With Γ = 0 it changes into the equation describing heat exchange in a liquid film in the absence of tangential forces at the liquid film—gas interface [2].

It is interesting to compare the results of the solution of the problem stated above by the method of integral equations [1] and by the proposed method. Such a comparison is made in Table 1 with the following values of the parameters: El = 1, E5 = 0.5, Pr = 10, h_0 = 0.3 cm, L = 100 cm, $\nu = 10^{-2}$ cm/sec; one prime and two primes denote the values for the method of integral equations [1] and for the proposed method [2], respectively. The average coefficient of heat transfer is calculated from the equation

TABLE 1. Comparison of Results of Solution of the Problem by Different Methods

Re	$\beta''_{av}/\beta'_{av}(\Gamma=0)$	$\beta''_{av}/\beta'_{av}(\Gamma=1)$
50	1,05	1,08
100	1,128	1,12
300	1,303	1,35
500	1,439	1,52
1000	1,731	1,88

$$\beta_{av}=\beta_{in}\frac{x_k}{L}+\beta_w\frac{L-x_k}{L},$$

where

$$x'_k=[(0.08E1+0.025)\,\mathrm{Re}+15E1+5]\,h_0,$$

$$x''_k=\left[\left(0.09E5+0.07\right)\mathrm{Re}+\frac{4.5}{E1E5}\right]\delta,$$

with the quantities β'_{in} and β_w being calculated from the equations [1]

$$\beta'_{in}=1.5\,\mathrm{Re}^{1/2}E5^{1/2}\nu^{1/2}a^{1/2}L^{-1}\exp\{2.3\sqrt{L}\exp(-0.511E1-1.236)+$$

$$+[(-0.045\Gamma1-0.45)\ln\mathrm{Re}+0.92]\,E5-0.22\ln\mathrm{Pr}+0.414E1-0.69\},$$

$$\beta_w=\left[0.82a^{2/3}q^{1/9}\omega^{4/9}A^{2/9}\left(1-\frac{\tau_0\delta}{6\mu q}\right)^{1/3}\right]L^{-1/3}\nu^{-2/3}\left[\exp\left(-\frac{0.26\tau_0}{\rho\omega^{4/3}A^{2/3}q^{1/3}\nu^{1/3}}\right)\right]^{-2/3}.$$

The difference in the results of the calculations by the two methods is explained in a way similar to that in [2].

NOTATION

a, coefficient of thermal diffusivity; ν, viscosity; ρ, density; h_0, initial thickness of liquid film; A, characteristic of Archimedes spiral; R(x), radius of curvature of spiral; P(x, y), hydrostatic pressure; H(x), equation of interface, determined from solution of the problem; T_w, T_{fi}, temperatures at wall of spiral and at film surface; $x = A(\theta\sqrt{\theta^2+1}+\ln|\theta+\sqrt{\theta^2+1}|)/2$, current length of spiral; $\mathrm{Re} = eq/\nu$, modified Reynolds number; $\mathrm{Ga} = \omega^2 A h_0^3 \nu^{-2}$, Galileo number; $E1 = \delta/h_0$, ratio of thickness of boundary layer to initial thickness; $E5 = h_0/A$, dimensionless characteristic of spiral; $\mathrm{Pr} = \nu/a$, Prandtl number.

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